ON THE ANALYSIS OF BODIES OF MINIMUM DRAG AT HYPERSONIC SPEEDS

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In PMM Vol.27, № 1, 1963 there was published an article by Gonor "On the form of three-dimensional bodies of minimum drag at hypersonic speeds" [1].

The question discussed in that paper, in somewhat more expanded form, made up part of a paper at a conference on variational problems in aerodynamics held in December, 1962 in the city of Seattle, U.S.A. (G.G.Chernyi and A.L.Gonor, "Determination of minimum drag bodies using Newton and Busemann pressure laws", BSRL Paper, Dec.1962).

These papers, and also the paper published by Gonor in this issue (see pages 471-475), led to a discussion in which A.Busemann, B.M.Bulakh, R.Jones, A.N.Kraiko, G.I.Maikapar, A.Miele, O.S.Ryzhov and W.Hayes took part, either during the presentation of the paper cited or in reviews of Gonor's article requested by the editor.

In view of the general interest, the results of this discussion are presented below, with editorial approval.

In the three papers cited, the problem of three-dimensional hypersonic flow of a perfect gas over bodies of minimum drag is formulated and solved for the first time.

In the solution the pressure on the surface of the body is determined approximately by Newton's formula. For the solutions obtained, the optimum bodies have a "star-shaped" cross-section; the drag of these bodies is appreciably lower than the drag of the equivalent cone, and can be made arbitrarily small by increasing the number of points.

Newton's formula is empirical in nature, and it is known from solutions of flows over bodies of revolution and airfoils, that on concave surfaces of such bodies (either smooth or having several corners, so that the flow goes through a system of compression shocks) the pressure can differ significantly from that determined by Newton's formula. Noting that fact, Busemann, Jones and Hayes pointed out that, in the optimum bodies obtained, the contour of the cross-section consists of concave sections, and, therefore, care should be exercised in using the results obtained; it is possible that the optimistic result showing a strong decrease of drag is connected mainly with an application of Newton's empirical formula beyond the range of validity. In remarks by Miele on the paper discussed in Seattle and by Maikapar on Gonor's first paper, doubt was also expressed as to the possibility of using Newton's formula in the problem considered. Maikapar referred to the results of his own work [2], in which he obtained an exact solution for supersonic flow over a class of pyramidal bodies with star-shaped cross-sections. The bodies found by Maikapar have appreciably lower drag than the equivalent circular cone. If the drag of these bodies is determined according to Neqton's formula and compared with the exact value, it is found that, for sufficiently large number of points, the ratio of the exact value to that determined by Neqton's formula increases without limit.

However this difference by itself is not sufficient to prove the inadmissibility of Newton's formula in the problem considered. Gonor explained the difference by observing that in the solution of Maikapar, for a sufficiently large number of points, the plane shock waves between the ridges of the pyramidal body correspond to one of the (two) possible solutions, namely the one which gives the larger pressure rise at the shock (Gonor considered this solution to be "physically incorrect"). Now Newton's formula, as is well known, can be used at high Mach numbers only as approximation to the second of the possible solutions, which gives a smaller pressure rise.

Later, in reviewing the paper by Gonor, which appears in this number of the journal, Maikapar expressed the opinion that it is possible to find an exact solution for supersonic flow over pyramidal bodies, using the plane flow surfaces which result from the straight intersection of two plane shock waves. Gonor has advised the editors that he was able, in fact, to determine such a solution. These solutions correspond to rather weak shock waves, and can be approximated sufficiently well by Newton's formula. A particular example of flow over a body with twenty points, computed by Gonor for $M = \infty$ and $\gamma = 1.4$ (γ is heat capacity ratio) showed that the exact value of drag of such a body is nearly 160 times lower than the drag of the equivalent circular cone; the error in using Newton's formula amounts to 16.5%. (We may note that, in the comparison, account should be taken of the concentrated forces which, in the Newtonian flow pattern, occur along the internal ridges of a pyramidal body).

Bulakh, with whose opinion Maikapar and Gonor were in agreement, believes that:

"both solutions, that of Maikapar and that of Gonor, are valid, but each one for certain conditions. The point is that, in the problem of flow over pyramidal bodies there are two solutions: the first one is characterized by a "strong" shock at the ridge of the body (it was obtained by Maikapar), and the second one by a "weak" shock. At high Mach numbers the latter is close to the solution according to Newtonian theory. Maikapar compared the Newtonian solution with the exact solution having a "strong" shock at the body ridge, and therefore, naturally, he obtained a large discrepancy, from which, however, one should not make conclusions as to the inadmissibility of Newton's formula, since the exact solution with weak shock should have been used in the comparison. On the other hand Maikapar's solution for a body with a large number of points, in our opinion, is correct. In fact, in that solution all gasdynamic equations and boundary conditions are satisfied, the flow after the plane shock wave is supersonic, and, therefore, the flow over a pyramidal body of finite length is possible. In fact that, in a plane perpendicular to the body ridge the flow after the shock is subsonic (i.e. the velocity component normal to the ridge is subsonic – G.Ch.), is not an indication of error in the solution of Maikapar, since the plane shock between two ridges of the body can be regarded not as the "intersection" of two plane shocks originating from the ridges, but as the result of the breakup of the shock wave created by the whole body (as occurs for a circular cone) into separate sections, as the speed of the body increases. Thus, there exist two regimes for flow over pyramidal bodies (theoretically)."

One can agree, on the whole, with these conclusions of Bulakh. It seems likely that, for supesonic flow of a perfect gas over semi-infinite conical bodies of arbitrary cross-section, the following alternatives exist: either there is no solution of conical character (i.e. one for which the parameters of the gas are constant along points from the cone vertex), or there are two such solutions.

Depending on the form of the body cross-section, the Mach number M and the values of γ, the conical bow wave in these two solutions may have either only a common vertex with that of the cone, or, in addition, if there are sharp ridges on the body, it may be attached to the ridges.

The exact solutions constructed by Maikapar and Gonor allow one to conjecture that, for bodies of pyramidal form with a sufficiently large number of points, two different flow regimes with the attached waves along the ridges are possible. In the simplest cases of flow over conical bodies as finite length cones, wedges, delta wings with the attached shock waves along the edges – it is well known that the flow regimes are always those with rather weak shock waves.

Thus, there are at present no conclusive arguments to exclude the use of Newton's formula for approximately determining the pressure on the surface of three-dimensional bodies in flow over with the attached shock waves. Of course, it would be desirable to have an estimate of the region of variation of the parameters of the problem for which the flow pattern does not differ greatly from that determined by using Newton's formula.

We note in closing that Equations (3) and (4) of [1] had been used earlier in [3]. In [1] it was also not stated that the reduction of drag of bodies with star-shaped cross-section compared to the equivalent circular cone had been given in [2 and 3].

BIBLIOGRAPHY

1. Gonor, A.L., O forme prostranstvennykh tel naimen'shego soprotivleniia pri bol'shikh sverkhzvukovykh skorostiakh (On the form of three-dimensional bodies of minimum drag at hypersonic speeds). *PMM* Vo.27, № 1, 1963.
2. Maikapar, G.L., O volnovom soprotivlenii neosesimmetrichnykh tel v sverkhzvukovom potoke (On the wave drag of nonaxisymmetric bodies in supersonic flow). *PMM* Vol.23, № 2, 1959.
3. Maikapar, G.L., O forme sverkhzvukovogo samoleta (On the form of a supersonic airplane). Trud.tsent.aero-gidrodin.Inst.Mosk. (TsAGI), № 841, 1961.

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